[illegible]:

$$g(z) = \frac{z^3+1}{z(z-i)^3} = \frac{1}{z} f(z)$$

DL $f(z)$ en 0 : $\dfrac{z^3+1}{(z-i-1)+1}$

Finalement

$$\int_{-\infty}^{\infty} \frac{1}{x} e^{ix}\, dx = i\pi$$

$$I = \int_{-\infty}^{\infty} \frac{\sin(x)}{x}\, dx = \operatorname{Im}\left(\int_{-\infty}^{\infty} \frac{1}{x} e^{ix}\, dx \right) = \pi$$

6.2.2 $\quad I = \displaystyle\int_0^{\infty} \frac{\cos(ax)}{1+x^4}\, dx \overset{\text{parité}}{=} \frac{1}{2} \int_{-\infty}^{\infty} \frac{\cos(ax)}{1+x^4} = \frac{1}{2} \operatorname{Re}\left(\int_{-\infty}^{\infty} \frac{e^{iax}}{1+x^4} dx \right) \quad a \geq 0$

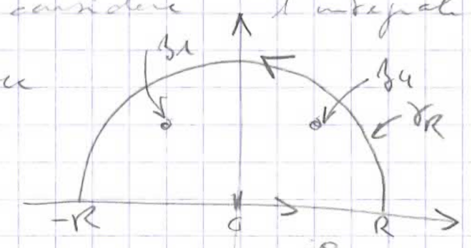
Racines de $P(z) = 1 + z^4 \Rightarrow (z_k)^4 + 1 = 0 \Rightarrow z_k^4 = -1$

$(z_k)^4 = e^{\pi i} \Rightarrow e^{4i\theta_k} = e^{i\pi} \Rightarrow 4\theta_k = \pi + 2k\pi \Rightarrow \theta_k = \frac{\pi}{4} + \frac{k\pi}{2}$

$z_k = e^{i\pi/4 + i\frac{k\pi}{2}} = \frac{\sqrt{2}}{2}(1+i) e^{i\frac{k\pi}{2}} = \frac{\sqrt{2}}{2}(\pm 1 \pm i)$

On considère l'intégrale : $\quad J = \displaystyle\int_{\gamma_R} \frac{e^{iaz}}{1+z^4} dz \qquad z_4 = \frac{\sqrt{2}}{2}(1+i)$

Avec $\qquad z_1 = \frac{\sqrt{2}}{2}(i-1)$



$$H_R^1 = \int_0^{\pi} \frac{e^{iaRe^{i\theta}}}{1+R^4 e^{4i\theta}} Rie^{i\theta} d\theta \qquad H_R^2 = \int_{-\infty}^{\infty} \frac{e^{iax}}{1+x^4} dx$$

On a : $\quad |H_R^1| \leq \displaystyle\int_0^{\pi} \frac{e^{-aR\sin(\theta)}}{1-R^4} R\, d\theta \underset{\text{(cf Exo précédent)}}{\leq} \frac{-R}{1-R^4} \frac{\pi}{aR} \left[e^{-a\frac{2R}{\pi}\theta} \right]_0^{\pi} \xrightarrow[R\to\infty]{} 0$

On calcule maintenant les résidus. On pose $f(z) = \dfrac{e^{iaz}}{\prod_{i=1}^{4}(z-z_i)}$

- $\operatorname{Res}(f, z_4) = \dfrac{e^{\frac{a\sqrt{2}}{2}(i-1)}}{(z_4-z_1)(z_4-z_3)(z_4-z_2)} = \dfrac{e^{\frac{a\sqrt{2}}{2}(i-1)}}{2\sqrt{2}(i-1)}$

$z_4 - z_1 = \frac{\sqrt{2}}{2} 2 = \sqrt{2}$

$z_4 - z_2 = \sqrt{2}(1+i)$

$z_4 - z_3 = i\sqrt{2}$

- $\operatorname{Res}(f, z_1) = \dfrac{e^{\frac{ia\sqrt{2}}{2}(i-1)}}{(z_1-z_4)(z_1-z_3)(z_1-z_2)}$